**ASSIGNMENT 1**

# PROBLEM 1

1. **List all possible functions f : {a, b, c} → {0, 1}**

There are 8 possible functions namely :-

* 1. f(a) = 0, f(b) = 0, f(c) = 0
  2. f(a) = 0, f(b) = 0, f(c) = 1
  3. f(a) = 0, f(b) = 1, f(c) = 0
  4. f(a) = 0, f(b) = 1, f(c) = 1
  5. f(a) = 1, f(b) = 0, f(c) = 0
  6. f(a) = 1, f(b) = 0, f(c) = 1
  7. f(a) = 1, f(b) = 1, f(c) = 0
  8. f(a) = 1, f(b) = 1, f(c) = 1

1. **Describe a connection between your answer for (a) and Pow({a, b, c})**

If S = Pow({a, b, c })

Then subset of set S are namely:-

1. {}
2. Pow{a}
3. Pow{b}
4. Pow{c}
5. Pow{a, b}
6. Pow{a, c}
7. Pow{b, c}
8. Pow{a, b, c}

We can clearly see that (a) and (b) both have same number of subsets/functions

1. **In general, if card(A) = m and card(B) = n, how many**
   1. **functions are there from A to B?**

Let A have a elements and B have b elements.

ba number of functions.

* 1. **relations are there between A and B?**

The total number of relations that can be formed between two sets is the number of subsets of their Cartesian product.

n(A) = p

n(B) = q

n(AXB) = pq

Number of relations between A and B is 2pq

* 1. **symmetric relations are there on A?**

Let A contains a elements

To be symmetric, whenever it includes a pair (x,y) (x,y), it must include the pair (y,x)

So A has a elements with (a/2) subsets of size 2.

Hence it has 2a/2  symmetric relations.

# PROBLEM 2

**For x, y ∈ Z we define the set: Sx,y = {mx + ny : m, n ∈ Z}.**

1. **Give five elements of S2,−3.**

For S2,-3

Set {2m -3n : m, n ∈ Z}

Let m = 0, n = 0 => 0

Let m = 1, n = 0 => 2

Let m = 0, n = 1 => -3

Let m = 1, n = 1 => -1

Let m = -1, n = -1 => 1

Hence five elements are:-

{0,2,-3,-1,1}

1. **Give five elements of S12,16.**

For S12,16

Set {12m + 16n : m, n ∈ Z}

Let m = 0, n = 0 => 0

Let m = 1, n = 0 => 12

Let m = 0, n = 1 => 16

Let m = 1, n = 1 => 28

Let m = -1, n = 1 => 4

Hence five elements are:-

{0,12,16,28,4}

**For the following questions, let d = gcd(x, y) and z be the smallest positive number in Sx,y.**

1. **Show that Sx,y ⊆ {n : n ∈ Z and d|n}.**

x = k1 . d

y = k2 . d (for some k1 andk2 ∈ Z)

We have

Sx,y = {mx + ny : m, n ∈ Z}

So mx +ny = m(k1 . d) + n(k2 . d)

d(m . k1 + n . k2)

Thus d | mx + ny which further implies, d | n

therefore Sx,y ⊆ {n : n ∈ Z and d|n }

1. **Show that {n : n ∈ Z and z|n} ⊆ Sx,y.**

n = k . z (for some k ∈ Z)

k . z ∈ Sx,y

k .z = k (mx + ny)

⇒ kmx + kny

⇒ ( km)x + (kn)y = n ∈ Z

therefore {n : n ∈ Z and z|n} ⊆ Sx,y

1. **Show that d ≤ z. (Hint: use (c))**

z = mx + ny

⇒ z = m (k1 . d) + n (k2 . d) (where x = k1 . d and y = k2 . d )

⇒ z = (m . k1 + n . k2) d

⇒ d = z / ( m . k1 + n . k2 )

Hence d ≤ z

1. **Show that z ≤ d. (Hint: use (d))**

d ∈ Sx,y

d = mx + ny

⇒ d = m (k1 . d) + n (k2 . d)

⇒ d = (m . k1 + n . k2) d

⇒ (m . k1 + n . k2) = 1 and ∈ Z

K1x + k2y = 1 (where x,y ∈ Z )

Hence z ≤ d

## PROBLEM 3

**We define the operation ∗ on subsets of a universal set U as follows. For any two sets.**

**A and B: A ∗ B := Ac ∪ Bc .**

**Answer the following questions using the Laws of Set Operations (and any derived results given in lectures) to justify your answer:**

1. **What is (A ∗ B) ∗ (A ∗ B)?**

( Ac ∪ Bc ) ∗ ( Ac ∪ Bc ) [using given A ∗ B := Ac ∪ Bc]

( Ac ∪ Bc )c ∪ (Ac ∪ Bc )c [using given A ∗ B := Ac ∪ Bc]

( ( Ac )c ∩ ( Bc )c ) ∪ ( ( Ac )c ∩ ( Bc )c ) [using de Morgan’s Law]

( A ∩ B ) ∪ ( A ∩ B ) [using Double complementation Law]

A ∩ B [using Idempotence Law]

1. **Express Ac using only A, ∗ and parentheses (if necessary).**

Ac = A ∗ A

Proof :

A ∗ A= Ac ∪ Ac [using given A ∗ B := Ac ∪ Bc]

(A − U) ∪ (A − U) [as we know Ac = A − U]

A − U [using Idempotence Law]

Ac [as we know Ac = A − U]

1. **Express ∅ using only A, ∗ and parentheses (if necessary).**

∅ = ( ( A ∗ A ) ∗ A ) ∗ ( ( A ∗ A ) ∗ A )

Proof:

Ø = Ø ∪ Ø [using Idempotence Law]

( A ∩ Ac) ∪ ( A ∩ Ac) [using Complementation Law]

( A ∪ Ac)c ∪ ( A ∪ Ac)c [using de Morgan’s Law]

( ( Ac )c ∪ Ac )c ∪ ( ( Ac )c ∪ Ac )c [using Double complementation Law]

( Ac ∗ A )c ∪ ( Ac ∗ A )c  [using given A ∗ B := Ac ∪ Bc]

( ( A ∗ A ) ∗ A )c ∪ ( ( A ∗ A ) ∗ A )c [using Ac = A ∗ A, from 2b]

( ( A ∗ A ) ∗ A ) ∗ ( ( A ∗ A ) ∗ A ) [using given A ∗ B := Ac ∪ Bc]

1. **Express A \ B using only A, B, ∗ and parentheses (if necessary).**

A \ B = (A ∗ ( B ∗ B ) ) ∗ ( A ∗ ( B ∗ B ) )

Proof:

A \ B = A ∩ Bc

( A ∩ Bc ) ∪ ( A ∩ Bc ) [using Idempotence Law]

( Ac ∪ B )c ∪ ( Ac ∪ B )c [using de Morgan’s Law]

( A ∗ Bc )c ∪ ( A ∗ Bc)c [using given A ∗ B := Ac ∪ Bc]

( A ∗ ( B ∗ B ) )c ∪ ( A ∗ ( B ∗ B ) )c [using Ac = A ∗ A, from 2b]

( A ∗ ( B ∗ B ) ) ∗ ( A ∗ ( B ∗ B ) ) [using given A ∗ B := Ac ∪ Bc]

# PROBLEM 4

**Let Σ = {a, b}. Define R ⊆ Σ∗ × Σ∗ as follows: (w, v) ∈ R if there exists z ∈ Σ ∗ such that v = wz.**

1. **Give two words w, v ∈ Σ ∗ such that (w, v) ∉ R and (v, w) ∉ R.**

w = a

v = b

here w, v ∈ Σ ∗

but (w, v) ∉ R and (v, w) ∉ R

as v ≠ wz where z ∈ Σ ∗



w = b

v = a

here w, v ∈ Σ ∗

but (w, v) ∉ R and (v, w) ∉ R

as v ≠ wz where z ∈ Σ ∗

1. **What is R←({aba})?**

f(w) = wz

v = wz

w = vz

w/z = vz/z

v = w/z

f←(w) = w/z (where z ∈ Σ ∗)

now solve for ‘w’ we get

R = { (ab,aba) , (a,aba) , (λ,aba) , (aba,aba) }

therefore

R←({aba})⇒ { aba, ab, a, λ }

1. **Show that R is a partial order**

For partial order a relation must be Reflexive, Anti-Symmetric and Transitive

1. **Reflexive**

if z = λ

so v = w

(w, w) ∈ R and (v, v) ∈ R

Hence the given relation is reflexive.

1. **Anti-Symmetric**

Proof:

(w, v) ∈ R and (v, w) ∈ R

(w, v) ⇒ v = wz1

(v, w) ⇒ w = vz2

⇒ w = wz1z2

⇒ w = w (z1 = z2 =λ)

Hence the given relation is anti-symmetric

1. **Transitive**

Proof:

(w, v) ∈ R and (v, q) ∈ R

(w, v) ⇒ v = wz1

(v, q) ⇒ q = vz2

⇒ q = wz1z2

⇒ q = wz (z1 = z2 = z)

Therefore (w,q) ∈ R

Hence the given relation is transitive

From the above results we can conclude that the given relation is a

Partial Order.

# PROBLEM 5

**Show that for all x, y, z ∈ Z:**

**If x|yz and gcd(x, y) = 1 then x|z.**

yz = kx (where k ∈ Z) and gcd(x, y) = 1

for y = 0

z = kx

⇒ x | z

for y ≠ 0

z = (kx) / y

z = kyx (where ky = k / y and ky ∈ Z)

⇒ x | z

Hence x|yz and gcd(x, y) = 1 then x|z.